Proceedings of Asia-Pacific Conference on Communications 2007

Selection combining of two amplify-forward relay branches with individual links experiencing Nakagami fading

Babu Sena Paul
Dept. of ECE
IIT Guwahati
Assam, India

R. Bhattacharjee
Dept. of ECE
IIT Guwahati
Assam, India

***Abstract*-Relay based communication has gained considerable importance in the recent years. In this paper we find out the end-to-end statistics of a two hop non-regenerative relay branch, with each hop being Nakagami-m distributed. A closed form expression for the statistics of the signal envelope at the output of a selection combiner in the destination node is also derived and compared with the simulation results. These statistics are useful in understanding the system performance in terms of bit error rate and outage probability.**

I. INTRODUCTION

Communication through cooperative relaying is emerging as an important technique in modern wireless systems. Wireless relaying allows mobile terminals to participate in the transmission of information, themselves not being the initial source or the final destination [1][2]. Diversity is used to mitigate the effects of fading and therefore increasing the reliability of radio links in wireless networks. The main idea of cooperative diversity schemes is to use relay nodes as virtual antennas to facilitate the communication of one source-destination pair. Potential application areas of cooperation diversity include advanced cellular architectures, mobile wireless ad-hoc networks, and other hybrid networks in order to increase coverage, throughput, and capacity to transmit to the actual destination or next relay.

Cooperation diversity systems can be broadly categorized as either non-regenerative or regenerative depending on the relay functionality [3]. In the former, the relay simply amplifies and forwards (A & F) the received signal, while in the latter the relay decodes, encodes, and forwards the received signal. The A & F mode puts less processing burden on the relay and, hence, is often preferable when complexity and/or latency issues are important as these relays are usual mobile terminals which are battery powered and has limited processing capacity. However, in a non-regenerative system, because of the presence of intermediate relay nodes, the statistics of the signal received at the destination depends on the channel conditions the signal experiences in the individual links. For properly utilizing the relay link in system design we require to understand the statistical behavior of such channels. Analysis of statistical behavior of relay channel has been a research area of considerable interest and recently some papers dealing with the methods of determining the statistics of such relay have appeared in the literature [3].

In this communication we first consider the statistics of two-hop amplify forward type relay channels where the individual link experiences Nakagami-m fading [4]. Nakagami fading model provides the flexibility of changing the individual link statistics by changing the parameter m. For $m=1$, we get the conventional Rayleigh fading model while by taking $m>1$, the channel is made to behave more like a Rician channel. We first determine the density function of the signal received in the destination through such a relay link. While performing analysis we assume the relay to be an ideal noise free repeater with unity gain. Such assumptions set the upper bound on the system performance. In practical systems, the performance will degrade when noise is present at the relay. Such degradation can be estimated through simulation studies. Next we consider the selection combining of two such two-hop-amplify-forward relay links where individual links are Nakagami-m faded. We derive the closed form expression for the density function of the received signal at the output of the selection combiner. The density function obtained analytically is compared for some specific m values with the simulated one. This paper thus provides an analytical frame work for determining the statistics of a two-hop-amplify-forward relay channels and the selection combining of two such relay channels when the individual links are Nakagami-m distributed.

The remainder of the paper is organized as follows. Section II presents the end-to-end channel statistics of a relay branch. In section III the probability density function of the signal amplitude at the output of the selection combiner is derived and compared with the simulation results. Finally, Section IV summarizes the main results of the paper.

II. END-TO-END CHANNEL STATISTICS OF A RELAY BRANCH

Fig.1 shows a typical relay based system without any diversity. The signal reaches the destination (D) from the source (S) via a relay node (R). $h_1(t)$ and $h_2(t)$ are the channel statistics between the S-R and R-D link respectively. As mentioned, we model $h_1(t)$ and $h_2(t)$ as Nakagami-m distributed so that various fading scenarios can be generated as particular cases of the generalized model. The probability density function (pdf) of the amplitudes of $h_1(t)$ and $h_2(t)$ may be written as

$$f_{R_i}(r_i) = 2\left(\frac{m_i}{\Omega_i}\right)^{m_i} \frac{r_i^{2m_i-1}}{\Gamma(m_i)} \exp\left(-\frac{m_i}{\Omega_i} r_i^2\right) \tag{1}$$

where, $r_i > 0$ & $i = 1,2$. $\Gamma(\bullet)$ is the gamma function, m_i denotes the m parameter of Nakagami-m distribution and $\Omega_i = E\left[r_i^2\right]$.

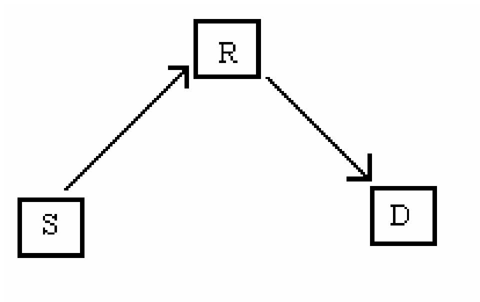

Fig. 1. A typical two hop relay based system.

R_1 and R_2 are the random variables representing the amplitudes of the links S-R and R-D respectively. m_1 and m_2 are the Nakagami-m parameters for the channels $h_1(t)$ and $h_2(t)$ respectively. In a typical 2-hop cooperative relaying environment, the source transmits the information in a time slot $\frac{T}{2}$ and the relay amplifies and retransmits the same information in another time slot $\frac{T}{2}$.For the flat fading case the received signal at the destination node may be written as,

$$y(t) = A(t)h_1(t)h_2(t)x(t) + A(t)h_2(t)n_1(t) + n_2(t) \tag{2}$$

where,
$x(t)$ is the transmitted signal
$A(t)$ is the gain of the relay
$n_1(t)$ and $n_2(t)$ are the additive noise at the relay and the destination respectively.

As we assume $A(t)=1$ and $n_1(t)=0$, the received signal at the destination can be written as,

$$y(t) = h_1(t)h_2(t)x(t) + n_2(t) \tag{3}$$

The effective channel between the source and the destination is basically the product of two random variables and can be written as $Z = R_1 \cdot R_2$. The density function of the random variable Z gives the channel statistics between the source and the destination via a relay. It has been shown in [5], that the density function of the product of N Nakagami-m random variables Y is given as,

$$f_Y(y) = \frac{2/y}{\prod_{i=1}^{N}\Gamma(m_i)} G_{0,N}^{N,0}\left[y^2 \prod_{i=1}^{N} \frac{m_i}{\Omega_i} \middle| \begin{matrix} - \\ m_1, m_2 \end{matrix} \right] \tag{4}$$

$G[\bullet]$ denotes the Meijer-G function and $\Gamma(\bullet)$ is the gamma function [6].

Specializing for a two hop system it can be shown that the density function of the channel between the source and the destination is given by

$$f_Z(z) = \frac{4}{z\Gamma(m_1)\Gamma(m_2)}\left(\frac{z^2 m_1 m_2}{\Omega_1\Omega_2}\right)^{\frac{m_1+m_2}{2}} K_{(m_1-m_2)}\left(2\sqrt{\frac{z^2 m_1 m_2}{\Omega_1\Omega_2}}\right) \tag{5}$$

where m_1 and m_2 are the Nakagami parameters of each hop and $K_\nu(\bullet)$ denotes the modified Bessel Function of second kind with order ν [6].

Fig.2. shows a comparison between the density function obtained analytically from (5) with the density function obtained from simulation of Nakagami-m distribution by employing techniques reported in literature [7]. The Nakagami parameters for the links were m_1 =1, m_2 =2, Ω_1 =2 and Ω_2 =4.

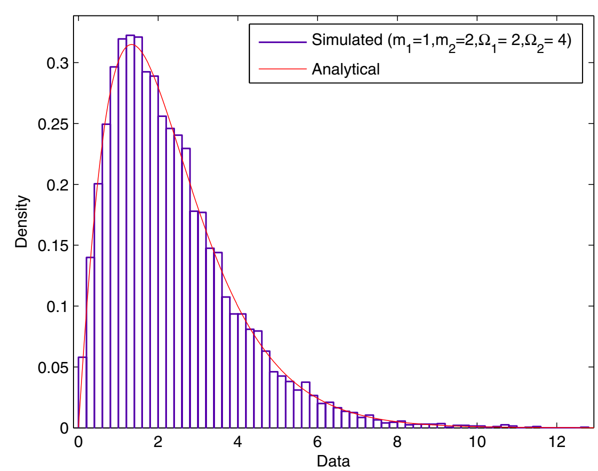

Fig.2. The S-D channel statistics of a two hop relay system

III. Two Branch Selection Diversity

The performance of a wireless link can be improved by applying combining techniques at the destination. Different combining techniques like selection combining, maximal ratio combining, equal gain combining, etc are generally used. In the present study selection combining of signals at the destination has been performed, using the statistics of the channels described in the earlier section. Fig. 3. shows the system model, containing two relays. Here we assume that the destination receives the signal only through the relays. The S-D link via R1 and that via R2 are assumed to be independent. $f_{R_1}(r_1)$ and $f_{R_2}(r_2)$ are assumed to be independent and gives the statistics of the S_D link via relay R1 and R2 respectively. The signal from the two relays are assumed to reach the destination in two orthogonal time slots, which are buffered for later processing.

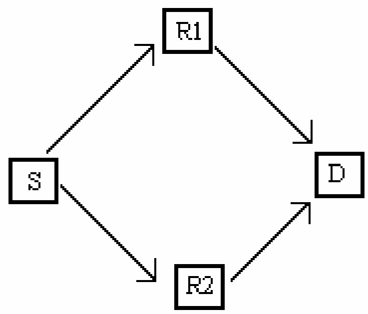

Fig. 3. Two branch dual hop relay diversity links.

The joint probability density function of two independent relay channels is

$$f_{R_1R_2}(r_1r_2)=\left[\frac{4}{r_1\Gamma(m_1)\Gamma(m_2)}\left(\frac{r_1^2m_1m_2}{\Omega_1\Omega_2}\right)^{\frac{m_1+m_2}{2}}K_{(m_1-m_2)}\left(2\sqrt{\frac{r_1^2m_1m_2}{\Omega_1\Omega_2}}\right)\right.$$

$$\left.\bullet\frac{4}{r_2\Gamma(m_3)\Gamma(m_4)}\left(\frac{r_2^2m_3m_4}{\Omega_3\Omega_4}\right)^{\frac{m_3+m_4}{2}}K_{(m_3-m_4)}\left(2\sqrt{\frac{r_2^2m_3m_4}{\Omega_3\Omega_4}}\right)\right] \tag{6}$$

The output of the selection diversity system has a signal-to-noise ratio equal to that of the strongest branch. The output signal-to-noise ratio will be that of branch 1, when it has the highest signal-to-noise ratio of both the branches. Under the condition that branch 2 has signal-to-noise ratio less than or equal to that of branch 1. Similarly, the output signal-to-noise ratio will be that of branch 2, when it has the highest signal-to-noise ratio and that of branch 1 is less than or equal to that of branch 1. Assuming equal average branch powers, when the envelope of the signal at the output of the combiner is the value s, the envelope of any one of the branches between branch 1 or branch 2 is also s and the envelope of the other branch is less than or equal to s. The occurrence that the envelope after selection combining is a value s, is the sum of all the occurrences over r_1 and r_2 that produce s as output. The new probability density function $f_S(s)$ describes the distribution of the output envelope of s, and is derived from the distribution of r_1 and r_2 following the approach given in [8]. Mathematically it can be written as,

$$f_S(s)=\int_0^s f_{R_1R_2}(r_1r_2)\Big|_{r_1=s}dr_2+\int_0^s f_{R_1R_2}(r_1r_2)\Big|_{r_2=s}dr_1 \tag{7}$$

$$f_S(s)=C\left[s^{m_1+m_2-1}K_{(m_1-m_2)}\left(2\sqrt{\frac{s^2m_1m_2}{\Omega_1\Omega_2}}\right)\bullet\left(\frac{1}{\alpha}\right)^{m_3+m_4}\int_0^{\alpha s}z^{m_3+m_4-1}K_{(m_3-m_4)}(z)\,dz\right.$$

$$\left.+s^{m_3+m_4-1}K_{(m_3-m_4)}\left(2\sqrt{\frac{s^2m_3m_4}{\Omega_3\Omega_4}}\right)\bullet\left(\frac{1}{\beta}\right)^{m_1+m_2}\int_0^{\beta s}t^{m_1+m_2-1}K_{(m_1-m_2)}(t)\,dt\right] \tag{8}$$

where,

$$C=\frac{16}{\prod_{i=1}^{4}\Gamma(m_i)}\left(\frac{m_1m_2}{\Omega_1\Omega_2}\right)^{\frac{m_1+m_2}{2}}\left(\frac{m_3m_4}{\Omega_3\Omega_4}\right)^{\frac{m_3+m_4}{2}}$$

$$\alpha=2\sqrt{\frac{m_3m_4}{\Omega_3\Omega_4}},\ \beta=2\sqrt{\frac{m_1m_2}{\Omega_1\Omega_2}}$$

Fig. 4. gives the plot of the density functions at the output of the selection combiner for the equation given in (8) as well for the density function obtained through simulation for the same parameters. The m-parameters of all the four links, of the two branches, were set to unity. $\Omega_1,\Omega_2,\Omega_3$ and Ω_4 were taken to be 2. The simulations were done in MATLAB. The Nakagami-m distributed random variables were generated following the procedure given in [7]. Selection at the receiver was done based on the signal strengths.

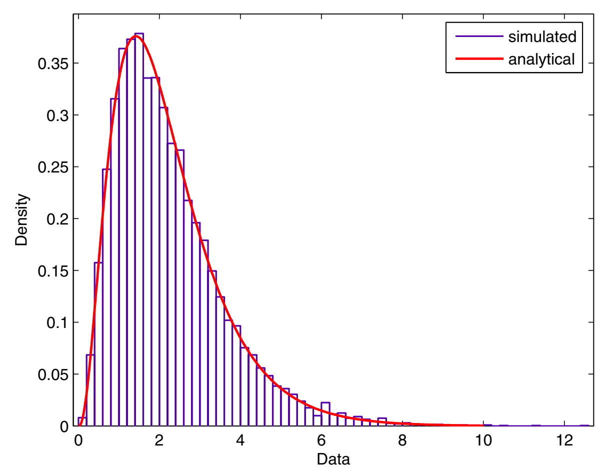

Fig. 4. Analytical and simulated density functions at the output of selection combiner

IV. CONCLUSION

In this communication we present the closed form expression for the probability density function of the signal envelope at the output of a selection combiner, having signals from two independent relay channels as inputs. The channel statistics of the individual hops of a relay diversity branch were assumed to be Nakagami-m distributed. We also evaluate the end-to-end density function of a two hop relay branch, with each hop being Nakagami-m distributed. The analytical results were verified through simulation for particular cases.

ACKNOWLEDGEMENTS

The authors would like to acknowledge the financial assistantship provided through project no. CSE/P/SN/4 (IIT Guwahati) for presenting the paper.

REFERENCES

[1] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity-part-I: system description," *IEEE Transactions on Communications,* vol. 51, no. 11, November 2003.

[2] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity-part-II: implementation aspects and performance analysis," *IEEE Transactions on Communications,* vol. 51, no. 11, November 2003.

[3] C. S. Patel, G. L. Stuber, T. G. Pratt, "Statistical properties of amplify and forward relay fading channels," *IEEE Transactions on Vehicular Technology,* vol. 55, no. 1, January 2006.

[4] M. Nakagami, ''The m-distribution—a general formula of intensity distribution of rapid fading,'' in *Statistical Methods in Radio WavePropagation*. Oxford, U.K.: Pergamon, 1960, pp. 3–36.

[5] G.K. Karagiannidis, N.C. Sagias, and P.T. Mathiopoulos, "The N* nakagami fading channel model," *2nd International Symposium onWireless Communication Systems*,September 2005.

[6] I.S.Gradshteyn ans I.M.Ryzhik, *Tables of Integrals,Series, and Products,* 5thed., San Diego, CA: Academic, 1994.

[7] N. C. Beaulieu, "Efficient nakagami-m fading channel simulation," *IEEE Transactions on Vehicular Technology,* vol. 54, no. 2, March 2005.

[8] K.Dietze, *Analysis of a Two-Branch Maximal Ratio and Selection Diversity System with Unequal Branch Powers and Correlated Inputs for a Rayleigh Fading Channel* ,MS Thesis, Virginia Polytechnic Institute and State University, 2001.